

# The University of the West Indies, Mona

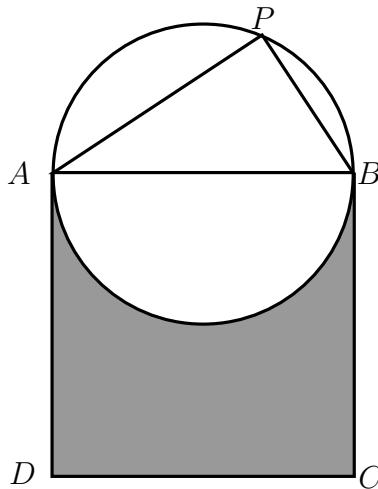
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## 2025-2026 Senior Mathematical Olympiad

Final Round Examination (Grades 7 and 8) - 11:00am

Provide complete solutions to all 10 questions (2.5 hours)

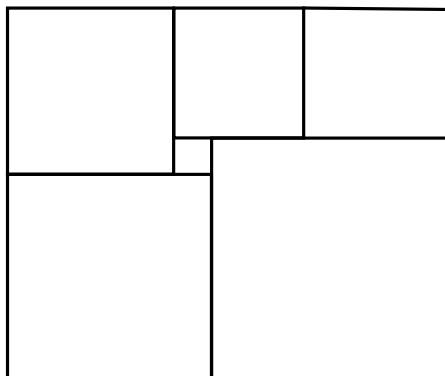
1. If  $a^4 = 9$ , determine the value of  $a^{10}$ .
2. In the diagram below,  $AB$  is a diameter of the circle and  $ABCD$  is a square. Given that  $AP = 12$  cm and  $PB = 16$  cm, determine the area of the shaded region, expressing your answer in the form  $a(b - c\pi)$ , where  $a, b$  and  $c$  are positive integers. What is the value of  $a + b + c$ ?



**Hint:** Triangle  $APB$  is right-angled.

3. The number of digits in the number 23 is two and the number of digits in 107 is 3. The total number of digits in the first  $n$  positive integers is 735. What is the value of  $n$ ?

4. The diagram below shows a rectangle that is divided into six squares. The smallest square has sides of length 1 unit.



Determine the perimeter of the rectangle.

5. Let  $x, y, z$  be real numbers with  $x \neq 0$  and  $y \neq 0$ . If

$$\frac{y}{x} = \frac{z}{y} = 2026,$$

determine the value of  $\frac{x+y}{y+z}$ .

6. Given that

$$a + \frac{1}{b + \frac{1}{c}} = \frac{101}{16},$$

where  $a, b$  and  $c$  are positive integers, show that  $a + b + c = 14$ .

7. Simplify the following expression:

$$\frac{\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \cdots \left(1 - \frac{1}{2026^2}\right)}{\left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{2026}\right)}.$$

8. A palindrome number is one that reads the same from left to right or from right to left. Two examples are 282 and 53335. How many 3-digit palindromes are there?
9. Four times the average of two different positive numbers is equal to three times the greater of the two numbers. The difference between the two numbers is three less than their average. What are the two numbers?
10. The triplet of positive integers  $(a, b, c)$  is called a *triple-triplet* if  $3a = 5b = 2c$ . Show that for every *triple-triplet*  $(x, y, z)$ , the product  $xyz$  must be divisible by 900.