The University of the West Indies, Mona

presents

2025-2026 Senior Mathematical Olympiad

Final Round Examination (Grades 7 and 8) - SOLUTIONS

1. **Soln**: Since $a^4 = 9$, $a^2 = 3$ and $a^{10} = a^4 \cdot a^4 \cdot a^2 = 9 \cdot 9 \cdot 3 = 243$

2. **Soln**: Using Pythagoras', $AB^2 = 12^2 + 16^2 = 400$ and therefore $AB = \sqrt{400} = 20$ cm. The area of half of the circle is $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi \times 10^2 = 50\pi$ and the area of the shaded region is $20^2 - 50\pi = 400 - 50\pi = 50\left(8 - \pi\right)$. The value of $a + b + c$ is $50 + 8 + 1 = 59$.

3. **Soln**: From 1 to 9, the number of digits is 9. From 10 to 99, the number of digits is $2 \times 90 = 180$. Therefore the number of digits from 1 to 99 is 189. From 100 to 199 the number of digits is $3 \times 100 = 300$. Since $735 - 189 = 546$ and $546 - 300 = 246$, the number of numbers from 200 is $\frac{246}{3} = 82$. That is 200 to 281 and so $n = 281$.

4. **Soln**: Let the dimensions of the squares be $1 \times 1, m \times m, (m+1) \times (m+1), (m+2) \times (m+2)$ and $(m+3) \times (m+3)$. Equating the lengths (or widths) of the rectangle, we have $m + m + m + 1 = m + 2 + m + 3$. This gives $m = 4$. The lengths and widths are therefore $3m + 1 = 13$ and $2m + 3 = 11$. The perimeter is $2\left(11 + 13\right) = 48$ units.

5. **Soln**: We have $z = 2026y$ and $y = 2026x$ and therefore

$$\frac{x+y}{y+z} = \frac{x+y}{2026x + 2026y} = \frac{x+y}{2026\left(x+y\right)} = \frac{1}{2026}.$$

6. **Soln**: The left hand side is $a + \frac{1}{\frac{bc+1}{c}} = a + \frac{c}{bc+1} = \frac{abc+a+c}{bc+1}$. Setting $bc + 1 = 16$ gives $bc = 15$ and $abc + a + c = 101$. So $15a + a + c = 101$. That is $16a + c = 101$. Because $bc = 15$, the possible values of c are $3, 5$. Since $101 = 16 \times 6 + 5$, we have $a = 6$ and $c = 5$. It follows that $b = 3$ and so $a + b + c = 14$.

   Alternatively: $\frac{101}{16} = 6 + \frac{5}{16} = 6 + \frac{1}{16/5} = 6 + \frac{1}{3+\frac{1}{5}}$ giving $a = 6, b = 3$ and $c = 5$.

7. **Soln**: Factorizing the numerator and cancelling,

$$\frac{\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots\left(1 - \frac{1}{2026^2}\right)}{\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\cdots\left(1 - \frac{1}{2026}\right)} = \left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\cdots\left(1 + \frac{1}{2026}\right)$$

This is

$$\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\cdot\cdots\cdot\frac{2026}{2025}\cdot\frac{2027}{2026}$$

After cancelling the result is $\dfrac{2027}{2} = 1013.5$

8. **Soln**: The palindromes take one of the following forms aba or aaa.

   Case aba: Conditionally, there are 9 choices for a and 9 choices for b. The total is $9 \times 9 = 81$

   Case aaa: There are 9 choices for the a. The total is 9

   The total is $81 + 9 = 90$.

9. **Soln:** Let ℓ and g be the two numbers where g is the greater of the two. From the information given,

$$4 \cdot \frac{\ell + g}{2} = 3g \text{ and } g - \ell = \frac{\ell + g}{2} - 3$$

   That is, $2(\ell + g) = 3g$ or $2\ell = g$ and $2(g - \ell) = \ell + g - 6$ or $g = 3\ell - 6$. We can now solve $2\ell = 3\ell - 6$. This gives $\ell = 6$ and $g = 12$.

10. **Soln**: A triple-triplet (x, y, z), takes the form $\left(t, \frac{3}{5}t, \frac{3}{2}t\right)$ or $(10a, 6a, 15a)$ where a is a positive integer. It is now evident that $xyz = 10a \times 6a \times 15a = 900a^3$ is divisible by 900.