Course Title: INTRODUCTION TO STOCHASTIC PROCESSES

Course Code: STAT6630

# of Contact hrs: One semester (13 weeks - 36 hours of lectures and 24 hours of tutorials)

Semester: II

No. of credits: 4

Prerequisites: None

RATIONALE:

Even though the students are familiar with the main notions of Probability Theory, they still need deeper and wider knowledge of many concepts of it. The Theory of Stochastic Processes will bring the students to a higher level of understanding of randomness, discussing how the random variables change over time, learning the exciting properties of Poisson process, Markov chain, Random walk, Brownian motion. This course will teach students to work successfully with stochastic modelling in different areas, including Finance, Queue Theory, Population Dynamics.

AIM:

This course aims to give students a broad overview of the main concepts in the theory of probability and stochastic processes. It will provide students with solid grounding in modern probabilistic and statistical methods.

LEARNING OUTCOMES:

By completion of this course, students should be able to:

- Define conditional expectation;
- Explain the idea of Bayers Estimators and apply it to the Insurance problems;
- Define the martingale process and solve selected problems about the martingale.
- State the Martingale Convergence Theorem and prove the Strong Law of Large Numbers;
- Define the Brownian motion process and apply it into Financial modelling;
- Define Markov chain process and apply it to the Birth and Death processes.

TEACHING METHODS:

Students will be exposed to the theoretical aspects of measure and integration via lectures. Tutorials and group problem solving sessions will be held regularly to ensure that students take an active role in the development of concepts encountered on the course, and are able to apply those concepts in the construction of mathematical proof.

The total estimated 48 contact hours may be accounted for as follows: 36 hours of lectures and 24 hours of tutorials. The tutorial periods will include problem solving sessions which will ensure that the students are able to understand, appreciate and apply the concepts learnt in the course. Course material, including note sets and exercises will be posted on the webpage http://ourvle.mona.uwi.edu/

SYLLABUS

Main notions of Probability Theory [8 hrs i.e. 6 hrs lecture + 4 hrs tutorials]

Probability, Random Variables, Borel-Cantelli Lemma, Expected value. Moment Generating and Characteristic Function, Laplace Transforms, Main Discrete and Continuous Probability

Distributions, Conditional Expectation and Bayes Estimators. The Exponential Distribution, Lack of Memory and Hazard Rate Functions. Some Probability Inequalities. Limit Theorems. Stochastic Processes.

Poisson Process: [8 hrs i.e. 6 hrs lecture + 4 hrs tutorials]
Counting Process, Poisson Process, Interarrival and Waiting Time Distributions, Conditional Distribution of Arrival Times.

Renewal Process: [6hrs i.e. 4 hrs lecture + 4 hrs tutorials]
Renewal Process , Some Limits Theorems, Wald's Equation, Elementary Renewal Theorem, Regenerative Process, Symmetric Random Walk and Arc Sine Laws.

Markov Chains: [10hrs i.e. 8 hrs lecture + 4hrs tutorials]

Markov Chains, General Random Walk, Simple Random Walk, Chapman-Kolmogorov Equation, Classification of States, Limit Theorems, Branching Processes. Continuous Time Markov Chains, Birth and Death Processes, Kolmogorov Backward and Forward Differential Equations.

Martingales: [8hrs i.e. 6 hrs lecture + 4 hrs tutorials]
Martingales, Stopping Times, Martingale Stopping Theorem, Azuma's Inequality for Martingales, Submartingales, Supermartingales, Martingale Convergence Theorem, Strong Law of Large Numbers.

Brownian Motion Process [8hrs i.e. 6 hrs lecture + 4 hrs tutorials]

Brownian Motion Process, Brownian Bridge Process, Hitting Times, Maximum Variable, Arc Sine Laws. Variations on Brownian Motion : Brownian Motion Adsorbed at a Value, Brownian Motion Reflected at the Origin, Geometric Brownian Motion, Integrated Brownian Motion, Brownian Motion with Drift. Using Martingales to Analyze Brownian Motion.

Total contact hrs = 48 hrs i.e. 36 hrs lecture + 24 hrs tutorials

ASSESSMENT:

The course assessment has two components consisting of coursework (40%) and final exam (60%)

One in-course test – 20% of overall grade;

One Written assignment – 20% of overall grade;

Final exam (3 hours) – 60% of overall grade.

REFERENCE MATERIAL

Text Books:

Ross: S. (2010). *Introduction to Probability Models*, 10th-edition, Academic Press. ISBN-10: 0123756863

Ross S. (1996). *Stochastic Processes.* Wiley Series in Probability and Statistics, 2-nd edition, 1996. ISBN-10: 0471120626

Brzezniak Z & T. Zastawniak (2002). Basic Stochastic Processes. A course through exercises.

Springer Undergraduate Series in Mathematics, Springer-Verlag, ISBN 2540761756

ONLINE RESOURCES:

1) Discrete stochastic processes_MIT OpenCourseWare

http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-262-discrete-stochastic-processes-spring-2011/index.htm

2) Course Notes for Stochastic Processes by Russell Lyons, based on the book by Sheldon Ross

http://mypage.iu.edu/~rdlyons/pdf/StochProc.pdf