Problem 1 (10 marks)
Find the general solution in terms of power series for the equation
\[(x^2 - 4)y'' + 3xy' + y = 0\]
around \(x = 0\). Then find a particular solution with \(y(0) = 4\) and \(y'(0) = 1\).

Problem 2 (15 marks)
Solve the initial value problem
\[(2x - x^2)y'' - 6(x - 1)y' - 4y = 0, \quad y(1) = 2, \quad y'(1) = 1\]
by the power series method. State the interval of values of \(x\) such that convergence is guaranteed.

**Hint**: first make a substitution of the form \(t = x - a\), then find a solution \(\sum_n c_n t^n\) of the transformed differential equation.

Problem 3 (6 marks)
Find all regular singular points of the differential equation
\[x^2(1 + x)y'' + x(4 - x^2)y' + (2 + 3x)y = 0.\]
What is the nature of the point at infinity?

Problem 4 (6 marks)
Find the Fourier series of the function \(f(t) = t\) with \(t \in [-2\pi, 2\pi]\).

Problem 5 (8 marks)
Show that the eigenvalues and eigenfunctions for the Sturm-Liouville problem
\[y'' + \lambda y = 0, \quad y(0) = 0 = y'(L)\]
are
\[
\lambda_n = \frac{(2n - 1)^2 \pi^2}{4L^2}, \quad y_n(x) = \sin \frac{(2n - 1)\pi x}{2L}, \quad n \geq 1.
\]
Problem 6 (8 marks)

Find the Fourier sine and cosine series of the function $f(t) = t(\pi - t)$ with $t \in [0, \pi]$. Which of the two series converges faster?