Problem 1 (10 marks)

A uniform slab of material with thermal diffusivity $k$ occupies the space region $0 \leq x \leq L$ and initially has temperature $T_0$ throughout the whole slab. Beginning at time $t = 0$, the endpoint $x = 0$ is held at temperature zero, while at the other endpoint $x = L$, heat exchange takes place with a surrounding medium at temperature zero so that $hT(t, L) + \partial_x T(t, L) = 0$, where $h$ is an appropriate heat transfer coefficient of the slab. Find the temperature $T(t, x)$ of the slab at any point $x$ and at any time $t$ such that $T(t, x)$ satisfies the boundary value problem

$$\partial_t T = k \partial_{xx} T,$$

$$T(t, 0) = 0,$$

$$hT(t, L) + \partial_x T(t, L) = 0,$$

$$T(0, x) = T_0.$$

Problem 2 (10 marks)

By considering the line integral of the vector field

$$\mathbf{F}(x, y) = y\hat{e}_x + (x^2 - x)\hat{e}_y$$

around the square in the $xy$-plane connecting the four points $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$, show that $\mathbf{F}$ cannot be a conservative field. You are not allowed to use the characterization of conservative vector fields in $\mathbb{R}^2$!