Problem 1 (10 marks)
Use the power series method to solve the initial value problem
\[ y'' + xy' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 0. \]

Problem 2 (15 marks)
Derive a three-term recurrence realtion for power series solutions of
\[ y'' + (1 + x)y = 0 \]
and find the first three nonzero terms in each of two linearly independent solutions.

Problem 3 (8 marks)
Find the Frobenius series solution of
\[ xy'' + 2y' - 4xy = 0. \]
around \( x = 0 \).

Problem 4 (6 marks)
Find the Fourier series of the function \( f(t) = t^2 \) for \( t \in [-1, 1] \).

Problem 5 (10 marks)
Consider the Sturm-Liouville problem
\[ y'' + \lambda y = 0, \quad y(0) = 0 = hy(L) - y'(L), \quad h > 0, \quad 0 < x < L. \]
Show that \( \lambda_0 = 0 \) is an eigenvalue if and only if \( hL = 1 \), in which case the associated eigenfunction is \( y_0(x) = x \).