Comments to the final exam

The grading was fair and if you lost any points on a question, it means that you missed something significant. If you got less than half of the full score on a question, it means that your answer was really wrong (not just poorly written). Perhaps the most important part of the learning process is understanding why a wrong answer is wrong. If after having read the solutions file to the final exam you cannot figure out what was wrong with your answer, ask me. The class average was 24 out of 60 and 28 out of 60 after removing the worst four scores. Nevertheless, 620014640 succeeded to get 41 out of 60. Finally, I do not believe that you can complain about the difficulty of the final exam since you received exact instructions about how you should have been prepared it.

The following table gives an overview of the overall performance in each question

<table>
<thead>
<tr>
<th>Question</th>
<th>Maximal marks</th>
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<tbody>
<tr>
<td>1</td>
<td>5/7</td>
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<tr>
<td>2</td>
<td>10/10</td>
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<tr>
<td>3</td>
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<td>4</td>
<td>5/13</td>
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<td>5</td>
<td>10/10</td>
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1. Question 1

(a) Most people wrote the proof in a blind way, that is they exchanged the role of the distance functions keeping the proofs for the different implications unaltered. This happens when you try to study things by heart without understanding the logic behind a proof.

(b) Many of you had no problem in proving boundedness. However, some people spoke about non existing concepts such as closed metrics or invented theorems that did not exist.

(c) Nobody was able to answer correctly to the last part of Question 1. All claimed that the real line was compact even though in the previous part you realized that the real line is actually clopen, i.e. closed and open.

2. Question 2

Some of you gave a clear and correct proof of the completeness of $\ell_\infty$. Some other people tried to put together some disconnected explanations or changed problem by proving completeness for the space $\ell_p$.

3. Question 3

Regarding Question 3 just one person did it well. The most typical errors you did are

(a) you took for $d$ the Euclidean metric, this is clearly not allowed since $d$ was from the very beginning an arbitrary metric.

(b) Instead of proving the triangle inequality for $d_\infty$ through the triangle inequality for $d$ you gave it for granted.
(c) Concerning the second part of Question 3 the most typical error was again dictated by a learning by heart approach to mathematics. You simply wrote a proof concerning bounded linear operators although Question 3 dealt with bounded mappings between metric spaces. Clearly, the two proofs have some common features but still they are different in their essence.

4. Question 4
Many of you applied in a wrong way the linearity property to the given operator in the problem. Most of you had in mind the right definition of a bounded linear operator but nobody was able to construct $\|Tf\|$ in a correct way. People forgot to take the norms of the coefficient functions $a$ and $b$ or started to speak about the diameter of $T$: a concept which does not exist. Finally, concerning the question about the invertibility of $T$ the most common mistake was a dimensional argumentation which applies if the underlying vector spaces are finite dimensional. Since the vector spaces in this problem were infinite dimensional the aforementioned approach was not acceptable.

5. Question 5
Some of you gave a clear and correct proof. Some other people started with “Let $(p_n(x))_{n \in \mathbb{N}}$ be a Cauchy sequence ...” and then they proved that the given sequence is Cauchy. Clearly, this is a logical mistake since the word “let” introduce the assumption that something has a certain property but then you do not need to prove that property. Actually, the right way to start would be “Let us consider a sequence $(p_n(x))_{n \in \mathbb{N}}$ of polynomials on $[0,1]$. We show that this sequence is Cauchy”.

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